

Lie Group and Representation Theory Seminar

Date: November 12 (Wed) , 2003, 18:00–19:00

Place: RIMS 402

Speaker: Christof Geiss, (Ciudad Universitaria, Mexico)

Title: Semicanonical bases and preprojective algebras

Abstract:

This is a report on joint work with J. Schröer and B. Leclerc.

Let \mathfrak{g} be a simple Lie algebra of type A, D, E and \mathfrak{n} a maximal nilpotent subalgebra of \mathfrak{g} . Moreover, let N be a maximal unipotent subgroup of a simple Lie group with Lie algebra \mathfrak{g} . Finally, let Π denote the corresponding preprojective algebra.

Lusztig's semicanonical basis \mathcal{S} of $U(\mathfrak{n})$ is parametrized by irreducible components of the corresponding nilpotent varieties $\text{mod}(\Pi, \mathbf{d})$. The dual \mathcal{S}^* is a basis of $\mathbb{C}[N]$.

We can show: For two elements of \mathcal{S}^* holds $b_C \cdot b_D \in \mathcal{S}^*$ if for the corresponding irreducible components holds $\text{ext}_{\Pi}^1(C, D) = 0$.

On the other hand, the dual canonical basis \mathcal{B}_q^* of $U_q(\mathfrak{n})$ specializes for $q = 1$ to a basis \mathcal{B} . We can show, that \mathcal{S} and \mathcal{B} have many elements in common, but \mathcal{B} and \mathcal{S} coincide only if Π is representation finite, i.e. in the cases $A_{2,3,4}$. This explains the multiplicative properties of the dual canonical basis observed previously in these cases.

On the other hand it gives us a good control over the dual semicanonical basis in the cases A_5 and D_4 , i.e. when Π is tame, since we have in this case a precise combinatorial description of the irreducible components of $\text{mod}(\Pi, \mathbf{d})$ in terms of indecomposable components. This is closely related to an elliptic root system of type $E_8^{(1,1)}$ resp. $E_6^{(1,1)}$.

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